A numerical investigation of bubble growth on and departure from a superheated wall by lattice Boltzmann method

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ABSTRACT

The bubble growth on and departure from a superheated wall has been simulated by an improved hybrid lattice Boltzmann method. The Briant's treatment of partial wetting boundary was introduced and the new hybrid model was validated by the single bubble growth on and departure from the superheated wall. The results showed that parametric dependencies of the bubble departure diameter were in good agreement with the experimental correlation from some recent literatures. This new model was also employed to simulate twin-bubble growth, coalescence on and departure from a horizontal superheated wall.

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1. Introduction

Nucleate boiling is a liquid–vapor phase-change process with the bubbles formation, growth, departure and rising. Because the process plays a key role in the boiling heat transfer, it has been widely studied for half a century. Although the phenomenon of bubble motion with its growth can be explained qualitatively as demonstrated in some experimental investigations, main difficulty in quantitative prediction is that multiphase flows are very complicated problems involving thermodynamics (co-existing phase), kinetics (nucleation, phase transitions) and hydrodynamics (inertial effects). Fortunately, the development of numerical methods and computer technology provides a powerful tool to predict vapor bubble behavior. For vapor bubble, the vapor–liquid interface becomes extraordinary complicated because of its nonlinearity and time-dependence behavior induced by phase-change with heat and mass transfer. Therefore, track of the interface is a key problem in the simulation of bubbly flows. Generally, the track can be classified as: the singular and the diffuse interface models. Earlier studies on vapor bubble dynamics were based on the Raleigh equation and its modification, which were basically dealt with zero or one dimensional problems [1]. Wittke and Chao [2] studied the collapse of a spherical bubble when it is in translatory motion. Cao and Christensen [3] simulated the bubble collapse in a binary solution by means of the transformed Navier–Stokes equation into the

stream function and vorticity in axisymmetric moving non-orthogonal body-fitted coordinates. Yan and Li [4] simulated a vapor bubble growth as it rises in a superheated liquid by two numerical methods proposed by Li and Yan [5,6]. Han and Seichi [7] used a mesh-free method to simulate bubble deformation and growth in nucleate boiling. Fujita and Bai [8] used the arbitrary Lagrange–Eulerian (ALE) method to simulate the growth of a single bubble on a horizontal surface with a constant contact angle. These numerical simulations are the singular interface model, in which the grid is limited so as not to be fit to bubble large deformation in topology, such as coalescence and breakup of the bubble. In such a situation, the diffuse interface model, such as VOF (volume of fluid), the level set and phase field method etc. was proposed to recover the defect. Tomiyama et al. [9] simulated a single bubble by the VOF. Hua and Lou [10] developed the front tracking method to simulate the bubble rising in the quiescent liquid. Son and Dhir [11] simulated a growing and departing bubble by the level set method. Ni et al. [12] simulated the bubbly flows with phase change by the level set method. Nevertheless, little information was reported about the vapor bubbles behavior with phase change and propagations of physical fields around a growing and deforming vapor bubble.

Recently, the lattice Boltzmann method (LBM) became a popular tool to simulate fluid flows due to its merits. Several LBM models were developed for multiphase and multi-component flows in the recent twenty years. The earlier one was the color method of Rothman and Keller [13], the potential method of Shan and Chen [14] the free energy method of Swift et al. [15] and the method

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Nomenclature

e	velocity vector of LBM	μ_ϕ	chemical potential
Eo	Eötvös number	ϕ	density difference (phase order parameter)
F	body force	τ	relaxation parameter
f	distribution function	θ_M	the mobility
g	distribution function	Γ	mobility coefficient
h_{fg}	latent heat of evaporation	ω	assigning parameter
Ja	Jacob number	λ	thermal conductivity
M	Morton number	δ	discrete size
n	total number density		
Pe	Peclet number		
t	time		
T	temperature in fluid field		
u	macroscopic velocity of the fluid		
V	volume of the bubble		
X	coordinates (x, y)		
ρ	density of fluid		

Subscripts/superscripts

i	discrete number
b	bubble
$G(g)$	gas
$L(l)$	liquid
eq	equilibrium

of He et al. [16]. These models have not been applied in practical problems owing to the limit of their small density ratio. Up to 2007, three models of large density ratio were proposed, they are model of Inamuro et al. [17], that of Lee and Lin [18] and that of Zheng et al. [19]. Inamuro's model was used to track the interface by applying a diffuse equation which is analogy to C–H (Cahn–Hilliard) equation. This technique is the same as the level set method. The interface is considered as an index such as 0 and 1. Unlike Inamuro's and Lee's models, Zheng used the more approximate C–H equation to track and define the interface without the artificial disposal. Benefited from the thought of an order parameter continuum in phase-change of Landau mean-field theory, Zheng's model can be extended into non-isothermal systems with phase-change. So, we proposed a hybrid LBM model [20], which is able to calculate the heat and mass transfer in multiphase flows through combining Zheng's multiphase and a thermal LBM models [21]. In this hybrid model, Zheng's model is added a source term in the corresponding C–H equation to define the phase-change and the thermal LBM is added a source term to define the latent heat. In the modified C–H equation, the treatment of the phase change at the interface can be explained clearly in physics and can make the interface be automatically tracked based on the change of the phase order parameter.

In this paper, the improved hybrid LBM is extended and Briant's treatment is employed to deal with the partial wetting boundary [22]. The new model is validated by simulating bubble growth and departure in the nucleate pool boiling. The numerical results can be considered as a basic work or a reference for generalizing LBM in the practical application about bubble flows.

2. Lattice Boltzmann model**2.1. Zheng's lattice Boltzmann dynamic model**

In Zheng's model, there are two independent macroscopic parameters, total number density, $n = \frac{\rho_A + \rho_B}{2}$ and number density difference, $\phi = \frac{\rho_A - \rho_B}{2}$, where ρ_A and ρ_B stand for the density of fluid A and B, respectively. n is proportional to pressure and approximately constant in the whole flow field. ϕ becomes positive in the region where $\rho_A > \rho_B$ and negative elsewhere, and thus it represents two-phase distribution as defined by Swift's model [15]. Two sets of discretized distribution functions f_i and g_i are used to assign each site, which are related to n and ϕ , respectively. f_i can be used to model the mass and momentum transfer, g_i can track the interface. Thus, the LBM BKG equation is

$$f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = \Omega_i \quad (1)$$

with

$$\begin{aligned} \Omega_i &= \frac{1}{\tau_n} [f_i(x, t) - f_i^{eq}(x, t)] \\ &+ \left(1 - \frac{1}{2\tau_n}\right) \frac{\omega_i}{c_s^2} \left[(e_i - u) + \frac{(e_i \cdot u)}{c_s^2} e^i \right] (\mu_\phi \nabla \phi + F_b) \delta t g_i(x) \\ &+ e_i \Delta t, t + \Delta t) - g_i(x, t) \\ &= (1 - q) [g_i(x + e_i \Delta t, t) - g_i(x, t)] - \frac{1}{\tau_\phi} [g_i(x, t) - g_i^{eq}(x, t)] \end{aligned} \quad (2)$$

where, x is sited on the lattice, and t is the time, τ_n and τ_ϕ are the dimensionless relaxation parameters. The equilibrium distribution functions to satisfy the conservation laws can be expressed as follows:

$$n = \sum_i f_i^{(eq)} \quad (3)$$

$$u = \frac{1}{n} \left[\sum_i f_i^{(eq)} e_i + \frac{1}{2} (\mu_\phi \nabla \phi + F_b) \right] \quad (4)$$

$$\sum_i f_i^{(eq)} e_{i\alpha} e_{i\beta} = (\phi \mu_\phi + c_s^2 n) \delta_{\alpha\beta} + n u_\alpha u_\beta \quad (5)$$

$$\sum_i g_i = \sum_i g_i^{(eq)} = \phi \quad (6)$$

$$\sum_i g_i^{(eq)} e_{i\alpha} = \frac{\phi}{q} u_\alpha \quad \text{with} \quad q = \frac{1}{\tau_\phi + 0.5} \quad (7)$$

$$\sum_i g_i^{(eq1)} e_{i\alpha} e_{i\beta} = \Xi_{\alpha\beta} \quad \text{with} \quad \Xi_{\alpha\beta} = \Gamma \mu_\phi \delta_{\alpha\beta} \quad (8)$$

where, u is the macroscopic velocity of the fluid. The chemical potential is given by,

$$\mu_\phi = A(4\phi^3 - 4\phi^2 \cdot \phi) - K \nabla^2 \phi \quad (9)$$

Performing a Chapman–Enskog expansion on Eqs. (1) and (2), the macroscopic equations for n and ϕ in the second order precision give

$$\frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0 \quad (10)$$

$$\frac{\partial (nu)}{\partial t} + \nabla \cdot (nuu) = -\nabla(P + \phi \mu_\phi) + \phi \nabla \mu_\phi + \nu \nabla^2 (nu) + F_b \quad (11)$$

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi u) = \theta_M \nabla^2 \mu_\phi \quad (12)$$

where, $\theta_M = q(\tau_\phi q - 0.5)\delta \Gamma$.

From Eqs. (10)–(12), the equilibrium distribution functions can be constructed as

$$f_i^{(eq)} = \omega_i A_i + \omega_i n \left(3e_{ix} u_x - \frac{3}{2} u^2 + \frac{9}{2} u_x u_{\beta} e_{ix} e_{i\beta} \right) \quad (13)$$

(Based on D2Q9)

where,

$$A_1 = \frac{9}{4} n - 15 \left(\phi \mu_{\phi} + \frac{1}{3} n \right) / 4,$$

$$A_{i(i=2,\dots,9)} = 3 \left(\phi \mu_{\phi} + \frac{1}{3} n \right),$$

$$\omega_1 = \frac{4}{9},$$

$$\omega_{i(i=2,\dots,5)} = \frac{1}{9},$$

$$\omega_{i(i=6,\dots,9)} = \frac{1}{36}.$$

$$g_i^{(eq)} = A_i + B_i \phi + C_i \phi e_i \cdot u \quad (\text{Based on D2Q5})$$

where, $B_1 = 1$, $B_i = 0 (i \neq 1)$, $C_i = \frac{1}{2q}$, $A_1 = -2\Gamma\mu_{\phi}$. Γ is the diffusion coefficient.

2.2. Inamuro's thermal LBM model

Inamuro et al. [21] proposed a model for the diffusion system in which there is the simplest distribution function h_i among other thermal models. The LBM equation is

$$h_i(x + e_i \Delta t, t + \Delta t) - h_i(x, t) = -\frac{1}{\tau_T} [h_i(x, t) - h_i^{eq}(x, t)] \quad (14)$$

where, τ_T is the dimensionless relaxation parameter.

The equilibrium distribution function (based on D2Q9) for the thermal model is

$$h_i^{eq}(x, t) = \omega_i T (1 + 3e_i \cdot u) \quad (15)$$

where, T is the temperature.

The diffusion equation corresponding the thermal model is

$$\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} = \delta \frac{1}{3} \left(\tau_T - \frac{1}{2} \right) \frac{\partial^2 T}{\partial x^2} \quad (16)$$

2.3. Phase change based on assumption of Stefan boundary

In the Landau mean-field theory, the phase change is considered as a continuous variable of order parameter. Thus, the C–H equation can be extended to include a phase-change term in the non-isothermal system. The phase change can be identified by the change of phase order parameter. Such a treatment can make the interface latent heat be tracked automatically. In addition, the phase-change latent heat is also considered in the LBM model. In order to simulate the vapor bubble growth on and departure from a superheated wall, the assumptions have to be made as:

Fig. 1. Effect of the different adsorb abilities when $\Phi_G < -90$ and $\Phi_G > -90$ on wetting boundary.

- (1) The vapor inside the bubble is pure and approximately incompressible;
- (2) The heat transferred to the interface makes the liquid completely evaporate into vapor for net increase of the bubble volume based on the Stefan boundary.

A vapor bubble of volume V'_b is introduced into a superheated liquid. In interval of $\Delta t'$, the mass transferred into the bubble is expressed as

$$\begin{aligned} \int_{V'} \frac{\Delta m}{\Delta t'} dV' &= \int \rho_G \frac{dV'_b}{dt} dV' = -\frac{1}{h_{fg}} \int_{S'} \lambda_l \left(\frac{\partial T'}{\partial x'} \right)_b dS' \\ &= -\frac{1}{h_{fg}} \int_{V'} \lambda_l \left(\frac{\partial^2 T'}{\partial x'^2} \right)_{V'} dV' \end{aligned} \quad (17)$$

where, ρ_G stands for vapor density, T' for temperature, h_{fg} for the latent heat and λ_l for thermal conductivity of liquid.

Based on the phase order parameter, the phase-change is considered as

$$\dot{\phi} = \frac{\Delta \phi}{\Delta t} = \frac{(\rho_L - \Delta m) - (\rho_G + \Delta m)}{2\Delta t'} - \frac{\rho_L - \rho_G}{2\Delta t'} = -\frac{\Delta m}{\Delta t'} \quad (18)$$

Eq. (17) is normalized by means of $V_b = \frac{V'_b}{V'_{bo}}$, $t = \frac{t'}{d_e U_T}$, $T = \frac{T' - T'_{\infty}}{T'_b - T'_{\infty}}$, $x = \frac{x'}{d_e}$, in which V'_{bo} is the bubble volume at an initial stage, d_e is the equivalent diameter of the bubble, U_T is terminal

Table 1
Comparison of the calculated radius of bubble growth with the Mikic's solution ($Pe_{lect} = 3000$).

Jacob/radius		Time				
		10	20	30	40	50
$Ja = 0.0006$	Mikic et al. [23]	0.04375	0.06375	0.07937	0.09324	0.10552
	Numerical data	0.04339	0.06158	0.0744	0.08519	0.09811
$Ja = 0.0009$	Mikic et al. [23]	0.06551	0.09243	0.1136	0.1303	0.14406
	Numerical data	0.06013	0.08691	0.10819	0.12704	0.14412
$Ja = 0.0012$	Mikic et al. [23]	0.08736	0.12382	0.15126	0.17416	0.19613
	Numerical data	0.08197	0.11746	0.14672	0.17236	0.19579

Fig. 2. Effect of gravity force and surface tension on bubble departure diameter.

rising velocity and T'_∞ is the temperature of liquid at the top boundary of the domain. So, the dimensionless form of the Eq. (17) is written as,

$$\rho_G \frac{dV}{dt} = -\frac{\lambda_l(T'_b - T'_\infty)}{h_{fg} U_T d_e} \left(\frac{\partial^2 T}{\partial x^2} \right) \quad (19)$$

By introducing the *Jacob* and *Peclet* numbers $Ja = \frac{1}{h_{fg}} C_{pl}(T'_b - T'_\infty)$ and $Pe = \frac{\rho_L U_T d_e C_{pl}}{\lambda_l}$, the Eq. (19) can be expressed as

$$\frac{dV}{dt} = -\frac{\rho_L}{\rho_G} \frac{Ja}{Pe} \left(\frac{\partial^2 T}{\partial x^2} \right) = \frac{\dot{\phi}}{\rho_L - \rho_G} \quad (20)$$

To include the phase change, the Eq. (2) when $\phi < 0$ is rewritten as

$$g_i(x + \vec{e}_i \Delta t, t + \Delta t) - g_i(x, t) = (1 - q)[g_i(x + \vec{e}_i \Delta t, t) - g_i(x, t)] - \frac{1}{\tau_\phi} [g_i(x, t) - g_i^{eq}(x, t)] + \omega_i \dot{\phi} \quad (21)$$

In the Eq. (16), $\delta \frac{1}{3} (\tau_T - \frac{1}{2}) = \frac{1}{Pe}$. So when $\phi < 0$, the latent heat term $\frac{\rho_G}{\rho_L(\rho_L - \rho_G)} \cdot \frac{\dot{\phi}}{Ja}$ can be added into the Eq. (14) and gives

$$h_i(x + \vec{e}_i \Delta t, t + \Delta t) - h_i(x, t) = -\frac{1}{\tau_T} [h_i(x, t) - h_i^{eq}(x, t)] + \omega_i \frac{\rho_G}{\rho_L(\rho_L - \rho_G)} \frac{\dot{\phi}}{Ja} \quad (22)$$

By the Taylor series expansion and the Chapman–Enskog expansion for Eqs. (21) and (22), the improved governing equations when $\phi < 0$ can be expressed in the second order

Fig. 3. Effect of bubble growth and adjustment Φ_L on departure diameter.

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi u) = \theta_M \nabla^2 \mu_\phi - \frac{\rho_L(\rho_L - \rho_G) Ja}{\rho_G} \left(\frac{\partial^2 T}{\partial x^2} \right) \quad (23)$$

$$\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} = \delta \frac{1}{3} \left(\tau_T - \frac{1}{2} \right) \frac{\partial^2 T}{\partial x^2} - \frac{\rho_G}{\rho_L(\rho_L - \rho_G)} \frac{\dot{\phi}}{Ja} \quad (24)$$

To validate the hybrid LBM model, a phase-change problem that can be solved analytically must be chosen. The test case is the bubble growth in a superheated liquid layer of infinite extent under the condition of no gravity. Before the bubble growing, a spherical bubble is rested in a superheated liquid layer. Grid of 100×100 is generated for the domain. From Table 1, it can be observed that the bubble growth shows a good agreement with the Mikic's analytical solution [23]. The comparison indicates that the treatment of the phase change based on the phase order parameter is feasible for the hybrid LBM model.

When the buoyancy force is considered, the Eo , M and Re are defined as

$$Eo = \frac{g(\rho_H - \rho_L)d^2}{\sigma}, \quad M = \frac{g(\rho_H - \rho_L)\mu_H^A}{\rho_H^2 \sigma^3}, \quad Re = \frac{\rho_H V_T d}{\mu_H}, \quad (25)$$

The bubble volume is calculated by means of $V'_b(t) = \sum_{\phi < 0} \phi(t) / \sum_{\phi < 0} \phi(t_0)$ and the growth rate of the bubble volume is calculated by $V'_b = [V'_b(t + \Delta t) - V'_b(t)] / \Delta t$.

2.4. The Briant's treatment of the partial wetting boundary

The Briant's treatment for the partial wetting boundary is introduced into the hybrid LBM model. The details can be found in [22].

Fig. 4. Bubble expanding and departure process at time interval of $\Delta t = 1500$ (steps).

Due to the particularity of vapor bubble departure, the adjustment of wetting boundary is considered as follows:

- (1) A little order parameter non-conservation induced by the distribution functions at inflow and outflow on the wetting boundary is counted and apportioned to every node occupied by the bubble.
- (2) The surface tension forces between the wall and fluids are adjusted to guarantee that the vapor bubble can grow on and depart from in integrality the wetting boundary like action of an actual vapor bubble in practical process. Therefore, according to the Young's law $[\cos\theta_w = (\sigma_{SG} - \sigma_{SL})/\sigma_{LG}]$, the order parameter Φ_G is set as -90 or smaller in this work. The contact angle is relatively adjusted with the corresponding Φ_L . The comparison of LBM test is shown in Fig. 1.

3. Numerical simulation of a vapor bubble growth on and departure from the superheated wall

In this simulation, $\rho_L = 1000$, $\rho_G = 1$, $\Gamma = 650$, $Pe = 3000$, $Eu = 72$ and $M = 3.44$. The mesh of the domain is generated as 100×50 . A spherical bubble with the radius of 3 is located in (50, 2). The domain is a rectangle with one partial wetting boundary (bottom wall), one extrapolated-boundary (top wall) and two stationary walls (left and right walls). In order to avoid the initial spontaneous

Fig. 5. Bubble diameters change with time.

diffusion, in all the following simulations, the initial fields can be considered as the initial temperature and force fields when the calculation time step t is bigger than 2000. It is because the interface spontaneous diffusion at this time can be approximately considered as disappearance. In addition, the initial thermal boundary layer thickness is calculated from the correlation [24]

$$\delta = 3/2 \frac{(T_w - T_\infty)R_c}{T_w - T_{sat}[1 - (2\sigma/R_c\rho_v L)]} \quad (26)$$

where R_c is the initial bubble radius.

As far as the bubble departure diameter (BDD) is concerned, the different physical parameters, such as body force, surface tension force, and partial wetting boundary and Ja are considered and investigated. The most widely used correlation for the BDD on the superheated surface was proposed by Fritz [25], in which the BDD was determined by a balance between the buoyancy and surface tension force acting normal to the solid surface. Based on his experimental measurement of the BDD, Staniszewski [26] modified the Fritz equation to obtain the BDD correlation as follows

$$D_d = 0.0071\beta \left(\frac{2\sigma}{g\Delta\rho} \right)^{\frac{1}{2}} \left(1 + 34.3 \frac{\partial D}{\partial t} \right) \quad (27)$$

where $\partial D/\partial t$ is the bubble growth rate.

Fig. 6. Propagation of flow fields with time.

Fig. 7. Propagation of temperature field with time.

Using the present method, the effect of physical parameters on the BDD is investigated as shown in Fig. 2. These figures show that the calculated BDD for the different gravity forces and surface tension forces are regressed to functions as $D \propto g^{-0.472}$ and $D \propto \sigma^{0.5}$. The results are in very good agreement with the Fritz relation. Fig. 3 shows the calculated correlation of BDD and the Ja , which is regressed a function as $D \propto Ja$.

The result shows that the correlation between the BDD and the bubble growth is the same as one of Staniszewski. Fig. 3 also shows that the BDD changes with the adjustment of Φ_L . Because the contact angle is determined by the Φ_L and Φ_G , the adjustment of Φ_L can change the contact angle and influence the BDD. The wetting boundary is set only for investigating the correlation among BDD

and the physical parameters such as the gravity force, surface tension force and growth rate. The results demonstrate that the boundary setting is feasible to the investigation.

3.1. The bubble growth on and departures from the partial wetted wall

With the present LBM, a vapor bubble growth on and departure from a heated and wetted wall has been calculated as shown in Fig. 4(a) and (b). It can be seen that the bubble is dilating on and departing from wetted wall. The base diameter is changed with time as shown in Fig. 5(a). The dilating and departing can be divided into two stages. At the first stage, associative action of the surface tension force dominates the dilating process because of

Fig. 8. Bubble growth and departure in different coalescence conditions.

the less volume of vapor bubble. It is also found that evolution of the vapor bubble at the initial process appears fluctuation which is identified with the Mukherjee's simulation [27]. At the second stage, the buoyancy dominates the departing process as shown in Fig. 4(b). It can be seen from Fig. 5(b) that the bubble growth with time conforms to $D \propto t^{0.448}$. The bubble growth under the influence of heated wall is lower than that with the proportion to the square root of the time in a superheated liquid layer of infinite extent.

3.2. Propagation of flow field

Fig. 6 presents the evolutions of flow field with the stream lines. It can be seen the influences of the bubble growth and departure on the flow field. At the initial stage, the bubble growth or expanding on the wetted wall induces two vortices to be formed on both sides of the bubble. The vortices are enforced to develop with further growing up and deforming of the bubble. With the process continuing, the deforming of the bubble results in the vertex breaking up into a twin-vortex. When the bubble starts with departing, the twin-vortices incorporate into a single vortex and rising up with the bubble. At the last stage, the vortices further strengthen their scopes and rise up accompanying with the bubble departure.

3.3. Propagation of temperature field

The propagation of temperature field is depicted in Fig. 7, from which it can be seen the effects of the bubble growth and departure on the temperature field. At the initial stage, because the bubble volume is small, the bubble growth is controlled by the heat transfer from both the micro and macro layer. With growing up of the bubble, the contribution of macro layer heat transfer is gradually weakened. In particular for the process of the bubble departure, the weakening tendency is more strength. It is because the forced convection induced by the ascending bubble makes the cold liquid far from bubble fill in space where the bubble occupied before its departure.

Fig. 9. Propagation of flow fields with time.

4. Characteristics of two bubbles growth on and departure from the wall

Based on the LBM scheme above, the mesh is set as 100×80 and the other setting is not changed. Two bubbles growth on and departure from the wall and coalescence dynamics are also investigated. The simulation is focused on the effect of coalescence in different twin-bubble distances on the bubble growth and departure as shown in Fig. 8. It is easily found that, when the twin-bubble distance (*dist*) is small, the bubble coalescence takes place so as to change the bubble growth rate. With the distance increasing, the coalescence is delayed and the departure time is shortened. But the BDD is not changed with the coalescence in the different twin-bubble distance like *dist* = 14, 16, and 17. With further increasing of the distance, the effect of coalescence on bubble growth rate is disappeared, but the BDD is increased like *dist* = 18 and 19. When the bubble departs from the surface in its integrity, the bubble growth rate tends to become zero so that its growth ceases.

Figs. 9 and 10 show the evolution of flow and temperature fields. In Fig. 9, before the bubble coalescence, the vortex is formed only on one side of the twin-bubble. With growing up and coalescence of the bubbles, the vortices are enhanced by the coalescence. The vortex on every side is split into one clockwise and one anti-clockwise vortex with their further growing up and ascending. Both vortices are developing further and converge into one vortex until its departure. Fig. 10 shows evolution of the

temperature field. It is easily found the forced convection directly influence the temperature field especially after bubble coalescence and departure.

5. Conclusions

The LBM multiphase model with a large ratio of density, combining with the LBM thermal model, is extended and developed into a hybrid model, which is able to predict the phase change process. In this paper, based on this model and used Briant's treatment of the partial wetting boundary, the bubble growth and departure on horizontal heated surface are investigated including the bubble coalescence. The numerical results exhibited correct parametric dependencies of the BDD as the experimental correlation in recent literature and showed the hybrid model is suitable. The results are as follows

- (1) Compared the numerical data with the related analytical and experimental results available in other literatures, it is found that the LBM simulation can correctly reflect the dynamics of the single bubble growth on and departure from the horizontal heated surface.
- (2) The temperature fields in nucleate pool boiling are directly simulated by means of the present hybrid model so that the heat transfer is quantitatively analyzed.

Fig. 10. Propagation of temperature fields with time.

- (3) Based on the LBM simulation, the effects of bubble coalescence on bubble growth and departure are also quantitatively investigated. The results can be made as a significant consult to the relative experimental and theoretical work.

Based on the analysis, the test calculation primarily demonstrated the LBM hybrid model capabilities of simulating bubble growth and departure in nucleate boiling. Due to the terseness advantage in the treatment of complex boundary, the LBM can be further extended to simulate the bubbly flow in special applications.

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